MATHEMATICS

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MATHEMATICS
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MATHEMATICAL REASONING

& Their Properties

CONTENTS

THINGS TO REMEMBER

✱ Intorducation

Logic is the subject which deals with the principles of reasonoing. Logic is some times defined as the science of proofs. Mathematics and other science subjects deal with the reasoning ans the arguments. Every students of mathematics and other science should know the principles of logice.

✱ Statement of Logical Sentence

We convey our daily views in the form of sentence which is a collection of words. This collection of words is called sentence, if it has some sence. Therefore,

"A declarative sentence, whose truth or falsity can be decided is called is called a statement of logical seentence but the sentence should not be imprevative, interrogative and exclamatory."

Statements are denoted by p, q, r,.....etc.

eg, "Delhi is the capital of India" is a statement, while "Do your work", is not a statement.

✱ Open Statement

A sentence which contains one or more variables such that when certain values are given to the variable it becomes a statement, is called an open statement.

eg, "He is a great man" is an open sentence because in this sentence "He" can be replaced by any person.

✱ Truth Value and Truth Table

Truth Value

A statement can be either "true" or "false" which are called truth value of a statement represented by T, F respectively.

eg, p : $\sqrt{2}$ is and irrational number.

q : Earth is flat.

In above statement, p is called (T) statement and q is called (F) statement.

Truth Table

A truth table is a summary of truth values of the resulting statements for all possible assignement of values to the variables appearing in a compound statement.

✱ Types of Statements

1. Biconditional Statement

Two simple statement connected by the phrase if and only if, form a biconditional statement. Symbol $\Leftrightarrow$ is used for "If and only If".

eg, The statement , ΔABC is isosceles if an only if AB = AC s a biconditional statement, written as

$$\Delta ABC \text{ is isosceles} \Leftrightarrow AB = AC.$$

If p, q are two statement, then the compound statement $(p \Rightarrow q) \wedge (q \Rightarrow p)$ is called a biconditional statement and is denoted by $p \Leftrightarrow q$ or $p \equiv q$.

2. Negative of Statement

A statement which is formed b changine the truth value of a given statement by using word like 'no' or 'not' is called negation o a given statement.

If 'p' is statement, then negation of 'p' is denoted by '$\sim p$'. The turth table for NOT is iven by

p	*~p*
T	F
F	T

Negation of a Biconditional Statement

$$\sim(p \Leftrightarrow q) \equiv \sim[(p \Rightarrow q) \wedge (q \Rightarrow p)]$$

$$\equiv \sim(p \Rightarrow q) \vee \sim(q \Rightarrow p)$$

$$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$$

$$\sim(p \Leftrightarrow q) \equiv \sim(p \wedge \sim q) \vee (q \wedge \sim p)$$

✷ **Logical Connectives or Sentencial**

Two or more statements are combined to form a compound statement by using symbols. These symbols are are called logical connectives.

Here, words and symbols are given to combine two sentences.

Word	Symbols
and	$\wedge$
or	$\vee$
implies that (if.....then)	$\Rightarrow$
if and only if (implies and is implied by)	$\Leftrightarrow$

✷ **Elementary Operation of Logic**

Formation of compound sentences from simple sentence using logical connectives are termed as elementary operation of logic.

There are three such operation discussed below

1. *Conjunction* A compound sentence formed by two simple sentences p and q using connective "and" is called the conjunction of p and q and represented by $p \wedge q$.

   eg, $p \wedge q \equiv$ Ramesh is a student and he belongs to Allahabad.

2. *Disjunction* A compound sentence formed by two simple sentences p and q using connective "or" is called the disjunction of p and q and is represented by $p \vee q$.

   eg, $p \vee q =$ *Bus left early or my watch is going slow.*

3. *Implication or conditional* A compound sentence formed by two simple sentences p and q

using connective "if.......then......" is called the implication of p and q and represencted by p q which is read as "p implies q". Here, p is called antecedent or hypothesis and q is called consequent or conclusion.

eg, $p \Rightarrow q \equiv$ if train reaches in time, then I can attend the meeting.

✷ Tautology and Contradiction

A compound statement is called a tatutology, if it has truth value T whatever may be the truth value of its compounds.

eg, Statement $(p \Rightarrow q) \wedge p \Rightarrow q$ is a tautology.

The truth table is prepared as follows

p	**q**	$p \Rightarrow q$	$p \Rightarrow q \wedge p$	$(p \Rightarrow q) \wedge p \Rightarrow q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Contradiction

A compound statement is called a contradiction, if it has truth value F whatever may be the truth value of its compounds.

eg, Statement $\sim p \wedge p$ is a contradiction.

p	$\sim p$	$\sim p \wedge p$
T	F	F
F	T	F

✷ Argument

An argument is a assertion that a given set of proposition $p_1, p_2, p_3, ...p_n$ called hypothesis, yields a proposition q, called the conclusion.

Such argument is denoted by

$$p_1, p_2, p_3 ....p_n \vdash q$$

The symbol $\vdash$ is called trunstile.

Validity of an Argument An argument

$$p_1 \wedge p_2 \wedge p_3 \wedge .... \wedge p_n \vdash q$$

is said to be valid, if q is true whenever each one of $p_1, p_2, p_3, ...p_n$ is true. This happens when $p_1 \wedge p_2 \wedge p_3 \wedge .... \wedge p_n \Rightarrow q$ is a tautonlogy.

Validity of an Argument

An argument, which is not valid is called a fallecy.

Rule of Inference

The argument which are universally valid are known a rule of inference.

- The argument $p \wedge (p \Rightarrow q) \vdash q$ is a valid argument.

(Law of detachment)

- For any statement p, q, r the argument

  $(p \Rightarrow q) \wedge (q \Rightarrow r) \vdash (p \Rightarrow r)$ is valid. (Law of syllogism)

- The arguments $(p \Rightarrow q) \wedge \sim q \vdash \sim p$ is valid
- The arguments $(p \Rightarrow q) \vdash (\sim q \Rightarrow \sim p)$ is valid.

(Law of contrapositive)

✷ Logical Equivalence

Two compound statements are said to be logically equivalent or tautologically equivalent, if then have the same truth values in all possible cases, ie, either both are true or both are false.

If the compound statements p and q are logically equivalent, then we writer $p \equiv q$.

Clearly, $p \equiv q$, if and only if $p \Leftrightarrow q$ is tautology.

✷ Algebra of Proposition

1. (a) $p \vee p \Leftrightarrow p$

   (b) $p \wedge p \Leftrightarrow p$ (Idempotent laws)

2. (a) $(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r)$

   (b) $(p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$ (Associative laws)

3. (a) $p \vee q \Leftrightarrow q \vee p$

   (b) $p \wedge q \Leftrightarrow q \wedge p$ (Commutative laws)

4. (a) $p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (q \vee r)$

   (b) $p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (q \wedge r)$ (Distributive laws)

5. (a) $p \vee F \Leftrightarrow p$

   (b) $p \vee T \Leftrightarrow T$ (Here, F is contradiction and T is tautology)

6. (a) $p \wedge T \Leftrightarrow p$

   (b) $p \vee T \Leftrightarrow T$

   (c) $p \wedge F \Leftrightarrow F$

7. (a) $p \vee \sim p \Leftrightarrow T$

   (b) $p \wedge \sim p \Leftrightarrow F$

8. (a) $p \vee (p \wedge q) \Leftrightarrow p$

   (b) $p \wedge (p \vee q) \Leftrightarrow p$ (Absorption laws)

9. (a) $\sim(p \vee q) \Leftrightarrow \sim p \wedge \sim q$

(b) $\sim(p \wedge q) \Leftrightarrow \sim p \vee \sim q$ (De-Morgan's laws)

✷ Law of Duality

Two formulae A and B are said to be duals of each other, if either one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

The connectives $\wedge$ and $\vee$ are also called duals of each other.

If a formula A contains the special variable T or F, then dual is obtained by replacing T by F and F by T in addition to the above mentioned inter-changes.

✷ Logical Inferences

To construct proofs, we need a mean of drawing conclusions or deriving new assertions from old ones. This is done by using rules or inferensece. Rules of inference specify whose conclusions may be inferred logitimately from assertions knwon, assume or previously established.

✷ Logical Implication

A proposition (statement) p logically implies a propersition q or q is a logical consequence of p, if te implication $p \to q$ is true for all possible assignment of the truth values of p and q that is $p \to q$ is a tautology.

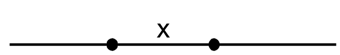
(in some books logical implication is represented as $A \Rightarrow B$.)

✷ Switching Circuits

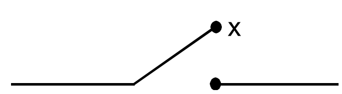
It is an arrangements of a finite number of switches, connected together along with a lamp between the two terminal of battery. The swithes are denoted by alphabets.

Switch is a device in an electric circuit which may have any of the two mutually exclusive states 'ON' and 'OFF'.

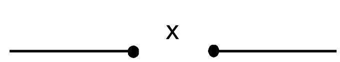
(a) ON state In the on state, the switch is closed and the current flows in the circuit and the lamp glows.



(b) OFF state In the on state, the switch is open and the current does not flows in the circuit and the lamp does not glows.



(c) VARIABLE state The switch x is the variable state is denoted by

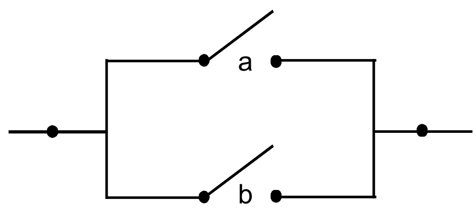


There are two basic ways in which switches are generally interconnected.

(i) In series (ii) In parallel

(i) Two switches a,b are said to be connected 'in series', if the current can pass only when both are in closed state and the current does not flow, if any one or both are open. The following diagram will show this circuit.

(ii) Two switches a, b are said to be connected 'in parallel', if current flows when any one or both are closed, and current does not pass when both are open. The following diagram will represent this circuit given by $a \vee b$.



If two switches in a circuit be such that both are open (closed) simultaneously, we shall represent them by the same letter. Again, If two switches be such that one is open, if the other is closed, we represent them by a and a'.

The value of a close switch or when it is on is equal to 1 and when it is open or off is equal to 0.

Operation on Circuits

Boolean Multiplication

The two switches r and s in the series will perform the operation of Boolean multiplication.

Clearly, the current will not pass from point S1 to S2 when either or both r, s are open. It will pass only when both are closed

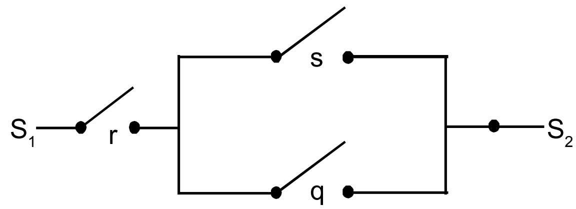
The truth table for $r \cdot s$ is given by

r	**s**	**r · s**
1	1	1
1	0	0
0	1	0
0	0	0

The operation is true only of the four cases ie, when both the switches are closed.

Bollean Addition

In the case of an operation of addition the two swithches will be in the parallel series as shown below.



The circuit show, that the current will pass when either or both switches are closed. It will not pass only when both are open.

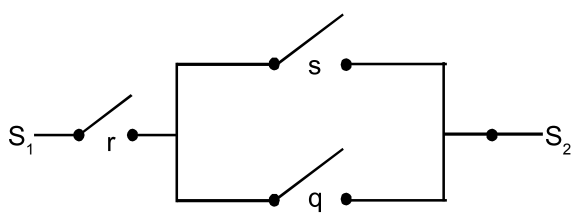
The truth table r + s is given by

r	**s**	**r + s**
1	1	1
1	0	1
0	1	1
0	0	0

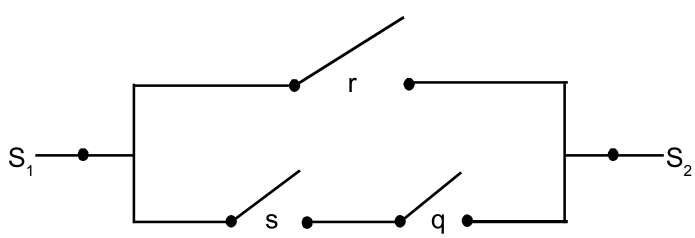
The operation is not true only in one of the four cases ie, when both r and s are open.

Circuits with Composite Operation

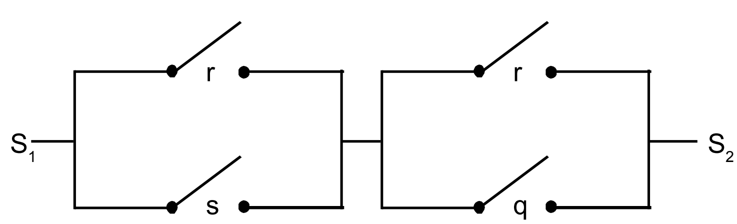
(i) Circuit for $r \vee (s \wedge q)$



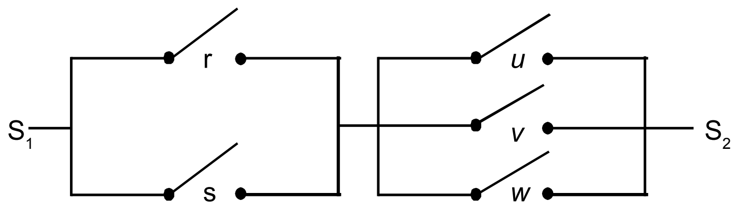
(ii) Circuit for $r \wedge (s \vee q)$



(iii) Circuit for $(r \wedge s) \vee (r \wedge q)$



(iv) Circuit for $(r \wedge s) \vee (u \wedge v \wedge w)$



Note :

- Simple statement which form a compound statement statement are called components.
- A statement is a neither a tautology nor a contradiction is a contigency.
- Understand that $A \rightarrow B$ is not same as $A \Leftrightarrow B$ ie, whenever there is $A \rightarrow B$ we cannot say B can logically inferred from A.
- If $A \leftrightarrow B$ is an equivalent statement (ie, $A \Leftrightarrow B$), then B can be logically inferred from A (ie, $A \Rightarrow B$).